[2]Turner, W.D. and Love, T.J., "Directional Emittance of a One-Dimensional Absorbing-Scattering Slab with Reflecting Boundaries," AIAA *Progress in Astronautics and Aeronautics: Thermal Control and Radiation,* Vol. 31, edited by C.-L. Tien, New York, 1973, pp. 389-395.

[3]Turner, W.D. and Love, T.J., "Directional Emittance from a Two-Dimensional Ceramic Coating," *AIAA Journal,* Vol. 9, Sept. 1971, pp. 1849-1853.

[4]Armaly, B.F., Lam, T.T., and Crosbie, A.L., "Emittance of Semi-Infinite Absorbing and Isotropically Scattering Medium with Refractive Index Greater than Unity," *AIAA Journal,* Vol. 11, Nov. 1973, pp. 1498-1502.

[5]Armaly, B.F., "Emittance of an Isothermal, Isotropically Scattering Finite Medium: Influence of Refractive Index," AIAA Paper 77-743; also published in AIAA *Progress in Astronautics and Aeronautics: Heat Transfer and Thermal Control Systems,* Vol. 60, edited by L.S. Fletcher, New York, 1978, pp. 151-170.

[6]Chandrasekhar, S., *Radiative Transfer,* Dover Publications, Inc., New York, 1960.

[7]Crosbie, A.L., "Emittance of an Isothermal, Isotropically Scattering Medium," *AIAA Journal,* Vol. 11, Aug. 1973, pp. 1203-1205.

[8]Crosbie, A.L., "Reflection Function for an Isotropically Scattering Finite Medium," *AIAA Journal,* Vol. 1., Oct. 1973, pp. 1448-1450.

Selective Optimal Orthogonalization of Measured Modes

Menahem Baruch*
University of Wisconsin-Milwaukee, Milwaukee, Wis.

In a Comment on Targoff's paper,[1] Rodden[2] argues that a method of orthogonalization which does not "corrupt" the rigid body modes[3] and assigns a higher credibility to the measurements of lower frequency modes[4] is preferable over a method in which all modes are treated equally. In his reply, Targoff[5] argues that for large structures it is difficult to simulate free-end conditions and the hypothesis that the measured modes have their errors apportioned in accordance to their modal frequencies is still unproven, especially for modes which occur in groupings with narrow frequency band.

Cannot both sides be right?

In this Note an optimal method is proposed in which the approach given in Ref. 6 is modified so that the rigid body modes are not corrupted and preference can be given, if desired, to groupings of mode shapes.

Let $R(n\times r)$ be a matrix which represents all measured or known mode shapes that have already been selected and orthogonalized. Hence

$$R'MR=I \tag{1}$$

where $M(n\times n)$ is a known symmetric positive definite mass matrix. In the beginning, R can represent the rigid body modes.

Let $T(n\times q)$ be a matrix which represents the group of measured modes which are now selected to be orthogonalized. It must be noted that the measured modes T_i have to be normalized in the following way:

$$T_i=\tilde{T}_i(\tilde{T}_i'M\tilde{T}_i)^{-1/2} \tag{2}$$

Received March 20, 1978; revision received Aug. 25, 1978.

Index category: Structural Dynamics.

*Professor, Mechanics Dept., College of Engineering and Applied Science; on sabbatical leave from the Department of Aeronautical Engineering, Technion, Haifa, Israel. Member AIAA.

where $\tilde{T}_i$ is the mode shape before normalization.

Following Ref. 6, we will look for a matrix $Q(n\times q)$ which satisfies the weighted orthogonality conditions

$$Q'MQ=I \tag{3}$$

and which minimizes the weighted Euclidean norm

$$\phi=\|N(Q-T)\|=n_{ij}(q_{jk}-t_{jk})n_{il}(q_{lk}-t_{lk}) \tag{4}$$

where the Einstein rule of summation is applied. $N(n\times n)$ is the positive definite symmetric solution of the relation

$$N=M^{1/2} \tag{5}$$

Now, in addition to constraint (3), Q must be also orthogonal to R:

$$Q'MR=0 \tag{6}$$

Note that Eq. (6) is a new constraint which does not appear in Ref. 6.

Using Lagrange multipliers to incorporate the constraints of Eqs. (3) and (6) into the cost function (4) the following Lagrange function is obtained:

$$H=\phi+\lambda_{il}(q_{ji}m_{jk}q_{kl}-\delta_{il})+2\beta_{is}q_{ji}m_{jp}r_{ps} \tag{7}$$

where δ_{il} is the Kronecker delta. β is a matrix of order $(q\times r)$ and Λ is a matrix of order $(q\times q)$. Due to the symmetry of Eq. (3), Λ must be symmetric

$$\Lambda'=\Lambda \tag{8}$$

The partial differentiation of Eq. (7) with respect to q_{fg}, where the results are equated to zero, yields equations that q_{fg} have to satisfy when H is minimal

$$\frac{\partial H}{\partial q_{fg}}=2n_{if}n_{il}(q_{lg}-t_{lg})+2\lambda_{gl}m_{fk}q_{kl}+2\beta_{gs}m_{fp}r_{ps}=0 \tag{9}$$

written in matrix form, Eq. (9) becomes

$$\frac{\partial H}{\partial Q}=2M(Q-T)+2MQ\Lambda+2MR\beta'=0 \tag{10}$$

Multiplication of Eq. (10) by R yields

$$\beta'=R'MT \tag{11}$$

It can be seen that β' represents the deviation from the orthonormality between the already orthogonalized modes R and the selection of measured modes T now treated.

By substitution of Eq. (11) into Eq. (10) one obtains

$$Q[I+\Lambda]=P \tag{12}$$

where $P(n\times q)$ is given by

$$P=T-RR'MT=[I-RR'M]T \tag{13}$$

Assuming that $I+\Lambda$ is invertable and using Eq. (3), one finally obtains

$$Q=P(P'MP)^{-1/2} \tag{14}$$

Note that for vanishing R Eq. (14) is identical to Eq. (14) of Ref. 6 where several techniques for its solution are described. It can be shown[6] that Q, obtained from Eq. (14) by using the positive square root of $P'MP$, minimizes the function H in Eq. (7).

Now, the technique for selective optimal orthogonalization of measured modes can be described as:

1) Select the rigid body modes and orthonormalize them in accordance with Eqs. (1) and (2) to obtain the R matrix.

2) Select the group of measured mode shapes with the highest measurement credibility. Normalize them in accordance with Eq. (2) to obtain T and then orthogonalize them in accordance with Eq. (14) to obtain Q.

3) Define a new R matrix which includes all the already orthogonalized modes ($[R_{\mathrm{new}}] = [R_{\mathrm{old}} \,\vdots\, Q]$).

4) Select a new group of measured mode shapes with lower measurement credibility, and so on.... Clearly, the selected group of measured modes may contain only one mode shape.

Using the technique just described one will obtain a weighted orthogonal matrix $X(n \times m)$ $(m \le n)$ in which the rigid body modes are not corrupted and the measurement credibility of the different groups of measured modes is incorporated by the order of their selection during the orthogonalization process

$$X^t M X = I \tag{15}$$

where X represents the orthogonalized measured modes.

Following Refs. 6 and 7, X can be used to obtain an optimally corrected stiffness matrix $Y(n \times n)$ from a given stiffness matrix $K(n \times n)$ [Ref. 6, Eq. (28), or Ref. 7, Eq. (23)].

$$Y = K - KXX^t M - MXX^t K + MXX^t KXX^t M + MX\Omega^2 X^t M \tag{16}$$

where Ω^2 $(m \times m)$ represents the measured frequencies which for the rigid body modes are zero. Note that Y incorporates the measured frequencies and their orthogonalized modes. Equation (16) can be used for further dynamic calculations.

Conclusions

A method was proposed by which the requirements of Rodden[2] or Targoff[5] of the orthogonalized measured modes can be satisfied in an optimal way. The supposedly known theoretical rigid body modes can be incorporated without corruption. If one agrees with Rodden[2] that the measurements of the lower frequency modes have a higher credibility, the measured modes must be incorporated into the proposed orthogonalization process one by one in the order of their ascending modal frequencies. Since every new corrected mode has to satisfy more constraints than the previous one, it is clear that the chance for a larger deviation between the measured and corrected modes will ascend for every newly selected mode. On the other hand, if one agrees with Targoff[5] that modes which occur in groupings with narrow frequency band have to be equally treated, one must incorporate the measured modes into the orthogonalization process, group by group. Clearly, all measured modes can be treated also as a single group.

Which way is preferable?

As long as there is no proven connection between the credibility of the measurements and their modal frequencies, this will be be a question of taste and intuition on the part of the engineer. However, the method of orthogonalization of measured modes proposed here gives the practicing engineer a tool whereby he can satisfy his taste and intuition in an optimal way.

References

[1]Targoff, W. P., "Orthogonality Check and Correction of Measured Modes," *AIAA Journal*, Vol. 14, Feb. 1976, pp. 164-167.

[2]Rodden, W. P., "Comment on 'Orthogonality Check and Correction of Measured Modes'," *AIAA Journal*, Vol. 15, July 1977, p. 1054.

[3]Rodden, W. P., "A Method for Deriving Structural Influence Coefficients from Ground Vibration Tests," *AIAA Journal*, Vol. 5, May 1967, pp. 991-1000.

[4]McGrew, J., "Orthogonalization of Measured Modes and Calculation of Influence Coefficients," *AIAA Journal*, Vol. 7, April 1969, pp. 774-776.

[5]Targoff, W. P., "Reply by Author to W. P. Rodden," *AIAA Journal*, Vol. 15, July 1977, pp. 1054-1056.

[6]Baruch, M. and Bar-Itzhack, I. Y., "Optimal Weighted Orthogonalization of Measured Modes," *AIAA Journal*, Vol. 16, April 1978, pp. 346-351.

[7]Baruch, M., "Optimization Procedure to Correct Stiffness and Flexibility Matrices Using Vibration Tests," *AIAA Journal*, Vol. 16, Nov. 1978, pp. 1208-1210.

Lifting-Line Theory of Oblique Wings in Transonic Flows

H. K. Cheng* and S. Y. Meng†

University of Southern California, Los Angeles, Calif.

I. Introduction

THE fluid dynamics of a high-aspect-ratio swept wing of practical interest is one characterized by its nonlinearity as well as its admission of the mixed (elliptic-hyperbolic) flow.[1,2] In the present work, we analyze three-dimensional (3-D) corrections to this nonlinear mixed flow by solving a perturbation problem and matching its solution to that of an outer flow. The latter is identified with a linear one involving a lifting line, similar to that in Prandtl and Van Dyke's works,[3,4] but the centerline of the planform is not required to be straight and unyawed, as was implicit in the classical theory.

Similar extensions of the lifting-line idea were made earlier in the context of unsteady and steady incompressible flows by Cheng.[5,6] Recently, we have carried out a corresponding development for transonic oblique wings involving nonlinear component flows in a manner consistent with the transonic small-disturbance approximation.[7] The theory treats high-aspect-ratio oblique wings as well as planforms with curved centerlines, also allowing symmetric swept wings. (The theory is inapplicable, of course, in the vicinity of the apex of a symmetric swept wing.) The main objectives of this Note are to show the existence of a similarity in the 3-D flow structure for certain oblique-wing geometry, and to demonstrate a solution to the reduced problem in a high-subcritical case.‡

In the following analysis the wing span is denoted by $2b$ and root chord by c_0; the aspect ratio $\mathcal{R}_l$ is defined as $\mathcal{R}_l \equiv 2b/c_0$. The wing camber and incidence are characterized by the parameter α, and the wing thickness ratio τ is assumed to be of $O(\alpha)$, or less. The sweep angle is Λ and the component Mach number is $M_n \equiv M_\infty \cos\Lambda$. The transonic similarity parameter for the basic component flow is $K_n \equiv (1 - M_n^2)/\alpha^{2/3}$. Emerging from the formulation is a reduced sweep angle $\Theta \equiv \Lambda/\alpha^{1/3}$ and the reduced aspect ratio $\epsilon \equiv 1/\alpha^{1/3}\mathcal{R}_l$. The analysis corresponds to the limit $\epsilon \to 0$ with fixed K_n and Θ. Only the domain $\Theta^2 \le K_n$ corresponding to a high subsonic, or a linear sonic, outer flow is studied.

Received July 5, 1978; revision received Sept. 6, 1978.

Index categories: Subsonic Flow; Transonic Flow; Aerodynamics.

*Professor, Dept. of Aerospace Engineering. Member AIAA.

†Research Assistant, Dept. of Aerospace Engineering.

‡Shock-free slightly supercritical examples are given in Ref. 7. Recently published works by Cook and Cole,[8] and Small[9] concern unyawed straight wings.